

An Improved Dilaton Background in Soft-Wall Model

Hongying Jin¹, Gang Liu²

Zhejiang Institute of Modern Physics, Zhejiang University, Hangzhou, China

Abstract

We adopted the form of v.e.v. of the bulk scalar field proposed by P.Zhang[10] in this paper, and changed the dilaton background from $e^{-\kappa^2 z^2}$ to $e^{-\kappa^2 z^2 + \alpha z}$ accordingly. The calculation shows that, as if we choose proper parameters, we can get a better result than both the present soft-wall models and the hard-wall model while keeping Regge behavior unchanged.

1. Introduction

So far there is no efficient way to perform analytical study on QCD in low-energy area, many efforts have been performed to approach this problem. Thanks to Maldacena's work[2], it is realized that the information about four-dimensional strongly coupled gauge theories can be extracted from gravitational theories in higher dimensions. The AdS/QCD, the efforts apply a five-dimensional theory on an anti-de Sitter (AdS) gravity background to learn something about QCD, is one of such approaches, which draws wide attention.

Although for QCD the exact form of the gravity dual is not yet known, the so-called bottom-up AdS/QCD approaches[5, 6] still give us deep impression. In the so-called hard-wall model[5], the authors introduce an infrared brane to make the theory confining, and thus the bulk space becomes a slice of AdS space. This slice of AdS space certainly matches QCD in the ultraviolet(UV) region due to QCD's asymptotic freedom. By choosing a "proper" cutoff, in ground hadron states, the hard-wall model fits the experiment well.

For the high excited states, the hard-wall model could not describe the Regge behavior[1]. In order to solve this problem, the so-called soft-wall model[1] was proposed, which replaces the "hard" cutoff with a "soft" cutoff. By introducing

¹Email: jhy@zimp.zju.edu.cn

²Email: G_Liu@zimp.zju.edu.cn

an exponentially decaying background $e^{-\Phi}$, the expected Regge behavior can be recovered. It has been shown[1] that, when the asymptotic behavior of Φ is z^2 as z goes to infinity, the Regge behavior may be recovered. The simplest choice of the Φ is $\kappa^2 z^2$, and the corresponding calculation shows that[6] it's much worse than the hard-wall model in the ground states. Some suggestions for improvement have been proposed[7, 8, 9, 10]. But generally these suggestions are not so satisfactory.

Actually, the form of the Φ may be determined by the self-interaction of the scalar field X [9]. In Ref.[9], where the self-interaction form is X^4 , both the UV and IR behavior of Φ is z^2 and so the original soft-wall model is appreciated. However in Ref.[10], where the self-interaction form is X^3 , the IR behavior of Φ is still z^2 while the UV behavior now becomes z . So in this work, we change the dilaton background $e^{-\kappa^2 z^2}$ to $e^{-\kappa^2 z^2 + \alpha z}$, while adopting the form of v.e.v of the scalar form introduced in Ref.[10]. By choosing proper parameters, we can fit the physical results very close to the experimental ones. The average error between the theory and the experiment can be reduced to 5.77%. Section 2 will explain our setup in detail, and we will show our results in section 3. In the last section we will take a discussion about our results.

2. Setup

In this section, we'll explain our modifications to the original soft-wall model in detail. The bulk space here is anti de Sitter space and the geometry takes the form of

$$ds^2 = \frac{1}{z^2}(-z^2 + \eta_{\mu\nu} dx^\mu dx^\nu), 0 \leq z \leq \infty \quad (1)$$

And the bulk action in the theory is

$$S = \int d^5x \sqrt{g} e^{-\Phi} \left\{ -\frac{1}{4g_5^2} (\|F_L\|^2 + \|F_R\|^2) + \|DX\|^2 - m_X^2 \|X\|^2 - \lambda \|X\|^3 \right\} \quad (2)$$

The $\|O\|^2$ signature means $\text{Tr}(O^\dagger O)$ here. The bulk scalar field X , which is dual to $\bar{q}q$ and thus is responsible for the chiral symmetry-breaking, obtain a 5D mass $m_X^2 = -3$ via the relation[3, 4] $(\Delta - p)(\Delta + p - 4) = m^2$. Every 5D field corresponds to a 4D p -form operator and Δ here is the dimension of the operator. We add a cubic term as Ref.[10] suggested in the action. The covariant derivative of X is $D^M X = \partial^M X - iA_L^M X + iA_R^M X$, where $A_{L,R}$ are

chiral gauge fields and $A_{L,R}^M = A_{L,R}^{Ma} t^a$ with $t^a = \sigma^a/2$ being the generators of the gauged isospin symmetry. The gauge fields part remains unchanged and we'll focus on the scalar part.

As in Ref.[6], the scalar field X can be divided into two parts, the v.e.v $X_0(z) = v(z)\mathbb{1}$ and the pion field π^a : $X = X_0(z)e^{2i\pi^a(x,z)t^a}$. Working in the axial-like gauge, $A_{L,Rz} = 0$, we can get the e.o.m. of v :

$$\partial_z \left(\frac{e^{-\Phi}}{z^3} \partial_z v \right) - \frac{e^{-\Phi}}{z^5} \left(\frac{3}{2} \lambda v^2 - 3v \right) = 0 \quad (3)$$

When $\lambda = 0$, which reduces to the case of Ref.[6] regardless of the dilaton form Φ , we find $v = m_q z + \sigma z^3$ in the UV region. However, in the IR region, this solution loses. Ref.[10] proposed one uniform form of $v(z)$:

$$v(z) = \frac{Az + Bz^3}{\sqrt{1 + C^2 z^2}} \quad (4)$$

The parameters are determined by following relations

$$m_q = \frac{2A}{\zeta}, \quad \sigma = 2\zeta \left(B - \frac{1}{2} AC^2 \right), \quad \gamma = \frac{2B}{C}, \quad \zeta = \frac{\sqrt{3}}{2\pi} \quad (5)$$

One can see that, when $z \rightarrow 0$, v behave as $(m_q \zeta z + \sigma z^3 / \zeta) / 2$ and when $z \rightarrow \infty$, $v \sim \gamma z^2 / 2$. According to the e.o.m., we can find the relation between the dilaton Φ and the v.e.v. v [12]:

$$\Phi'(z) = \frac{z^3}{v'} \left[\left(\frac{v'}{z^3} \right)' - \frac{1}{z^5} \left(\frac{3}{2} \lambda v^2 - 3v \right) \right] \quad (6)$$

We find that, in IR region, $\Phi \sim z^2$ which is just the requirement of the Regge behavior, and in the UV region, $\Phi \sim z$. So, we simply modify the dilaton background to $\Phi = \kappa^2 z^2 - \alpha z$.

Adding an αz term is not just a mathematical game. Let's study the e.o.m. of the vector-like gauge field $V = A_L + A_R$:

$$\partial_z \left(\frac{e^{-\Phi}}{z} \partial_z V_{\mu n}^a \right) + \frac{m_n^2 e^{-\Phi}}{z} V_{\mu n}^a = 0 \quad (7)$$

We can denote $\psi_n = \left(\frac{e^{-\Phi}}{z} \right)^{1/2} V_n$. We have dropped the subscript μ out for convenience. We can thus rewrite the e.o.m. into the Schrödinger form:

$$-\psi_n'' + V(z)\psi_n = m_n^2 \psi_n \quad (8)$$

Here,

$$V(z) = \kappa^4 z^2 - \kappa^2 \alpha z - \frac{\alpha}{2z} + \frac{3}{4z^2} - \frac{\alpha^2}{4} \quad (9)$$

Actually, Eq.(8) is more like a Klein-Gordon equation rather than a Schrödinger equation. Thus, $V(z)$ in Eq.(8) is somewhat like the square form of the potential in the corresponding Schrödinger equation. In quark model, the potential of the Schrödinger equation usually contains a linear potential, a Coulomb potential and vacuum energy. If such a potential writes like

$$V_S(z) = az - \frac{b}{z} + c \quad (10)$$

We find that, its square form is

$$\tilde{V}(z) = a^2 z^2 + (2ac)z - \frac{2bc}{z} + \frac{b^2}{z^2} + c^2 - 2ab \quad (11)$$

And such a potential is very similar to Eq.(9).

3. Results

Now we present our numerical results for QCD observables. There are four parameters in our model: κ , α , γ and the mass of quark m_q . The decay constant of pion f_π and the mass of pion m_π is fixed by experiment which are 92.3MeV and 139.6MeV separately. σ can be determined by m_π , f_π and m_q through Gell-Mann-Oakes-Renner relation $m_\pi^2 f_\pi^2 = 2m_q \sigma$ where m_q is chosen in the region given by the Particle Data Book. The values of the other three parameters are chosen in order to fit the experimental results of QCD observables, and these values are:

Parameter	κ	α	γ	m_q
Value(MeV)	609.6	777.3	49.8	3.29

Table 1: Parameters Chosen in the Model.

The results of QCD observables are exhibited in Tab.2. We find that, all results in ground state fit experiment very well except m_{a_1} , and the average error is about 5.77%, which is much better compared with existing results. We also plot the electromagnetic pion form factor $F_\pi(Q^2)$ in the spacelike region below, and compare our results to the hard-wall model. One can see that our result is

Observable	Experimental Value	Theoretical Value	Error(%)
m_ρ	775.5 ± 0.4	775.5	0
m_{a_1}	1230 ± 40	1476	20
$f_\rho^{1/2}$	346.2 ± 1.4	344.7	0.43
$f_{a_1}^{1/2}$	433 ± 13	465	7.4
$g_{\rho\pi\pi}$	6.03 ± 0.07	6.09	1

Table 2: Predictions for QCD observables, all experimental and theoretical values are in MeV, except $g_{\rho\pi\pi}$.

better, and fits the experiment quite well. Also, we can calculate the value of the mean pion charge radius squared through[11]

$$\langle r_\pi^2 \rangle = -6 \left. \frac{dF_\pi(Q^2)}{dQ^2} \right|_{Q^2=0} \quad (12)$$

The result is $0.456 fm^2$, which is consistent to the data from PDG[13] $0.45 fm^2$.

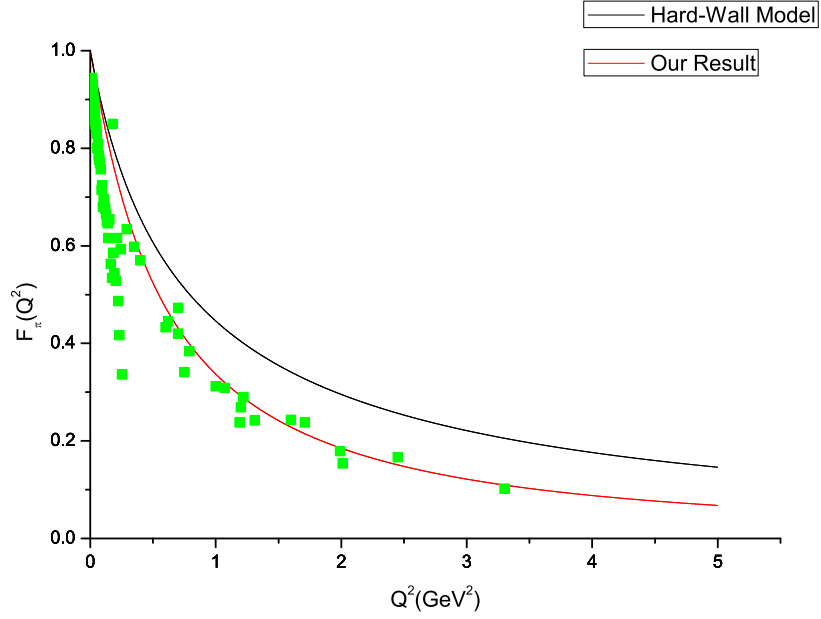


Figure 1: The electromagnetic pion form factor $F_\pi(Q^2)$ in the spacelike region. The blue plots are experimental results from Ref.[14].

4. Discussion and Conclusion

The hard-wall model can get a satisfactory ground state result while violating the Regge behavior and the original soft-wall model can achieve the Regge behavior, but the ground state result is not so satisfactory. In this work, we reanalyzed the soft-wall model and proposed a new dilaton background. With these modifications, we get a better result than the hard-wall model while keeping Regge behavior unchanged.

Honestly, the mass spectrum for both ρ and a_1 is not so satisfactory. We list our result of ρ here in comparison with the simplest soft-wall model in Ref.[6], the model in Ref.[9] and the model in Ref.[10] in Tab.3.

ρ	0	1	2	3	4	5
m_{exp}	775.5	1465	1570	1720	1900	2150
m_1	778	1100	1348	1556	1740	1906
m_2	475	1129	1429	1674	1884	2072
m_3	952.6	1349	1653	1909	2134	2338
m_{ours}	775.5	1376	1795	2149	2453	2715

Table 3: Mass spectrum results. m_{exp} are experimental results[13], m_1 are results from Ref.[6], m_2 are from [9], m_3 are from Ref.[10], and m_{ours} are our results. The average error of our results is 16.8%.

The reason for the disappointing mass spectrum is probably that the model is too simple. Things are not difficult to find a group of parameters that fit the total mass spectrum, but then the ground state will not so satisfactory just like Ref.[10] whose ground state error is 22.8%. Whether there is a model that can get satisfactory results for both ground state and the mass spectrum remains unknown.

Acknowledgement

This work is supported by NNSFC under Project No. 10775117. We would like to thank Prof. Jialun Ping for helpful discussions on numerical calculation.

References

- [1] A.Karch, E.Katz, D.T.Son and M.A.Stephanov, Phys. Rev. D74, 015005 (2006)[arXiv:hep-ph/0602229]
- [2] J.M.Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998)[Int. J. Theor. Phys. 38, 1113 (1999)][arXiv:hep-th/9802109]
- [3] E.Witten, Adv. Ther. Math. Phys. 2, 253 (1998)[arXiv:hep-th/9802150]
- [4] S.S.Gubser, I.R.Klenanov and A.M.Polyakov, Phys. Lett. B428, 105 (1998)[arXiv:hep-th:9802109]
- [5] J.Erlich, E.Katz, D.T.Son and M.A.Stephanov, Phys. Rev. Lett.95, 261602 (2005)[arXiv:hep-ph/0501128]
- [6] H.J.Kwee and R.F.Lebed, JHEP 0801, 027 (2008)[arXiv:0708.4054]
- [7] J.P.Shock, F.Wu, Y.L.Wu, and Z.F.Xie, JHEP 0703, 064(2007)[arXiv:hep-ph/0611227]
- [8] Y.Q.Sui, Y.L.Wu, Z.F.Xie and Y.B.Yang, Phys. Rev. D81, 014024 (2010)[arXiv:0909.3887]
- [9] T.Gherghetta, J.I.Kapusta, T.M.Kelley, Phys. Rev. D79, 076003 (2009) [arXiv:0902.1998]
- [10] P.Zhang, JHEP 1005, 039 (2010)[arXiv:1003.0558]
- [11] S.J.Brodsky and G.F.de Téramond, Phys. Rev. D77, 056007 (2008)[arXiv:0707.3859]
- [12] J.I.Kapusta and T.M.Kelley, Int. J. Mod. Phys. A25, 453 (2010)[arXiv:0908.0725]
- [13] K. Nakamura et al. (Particle Data Group), J. Phys. G37, 075021 (2010)
- [14] P.Brauel, Phys. Lett. B69, 253 (1977); C.J.Bebek, Phys. Rev. D17, 1693 (1978); H.Ackermann, Nucl. Phys. B137, 294 (1978); P.Brauel, Z. Phys. C3, 101 (1979); S.R.Amendolia, Phys. Lett. B138, 454 (1984); Nucl. Phys. B277, 168 (1986); T.Horn, Phys. Rev. Lett. 97, 192001

(2006)[arXiv:nucl-ex/0607005]; V.Tadevosyan, Phys. Rev. C75, 055205
(2007)[arXiv:nucl-ex/0607007]